# QUANTUM COSMOLOGY IN SOME SCALAR-TENSOR THEORIES

L.O. PIMENTEL†, C. MORA†‡

†*Departamento de Física, Universidad Autónoma Metropolitana, Apdo. Postal 55-534, CP 09340, México DF, México.*

‡*UPIBI-Instituto Politécnico Nacional, Av. Acueducto s/n Col. Barrio La Laguna Ticomán, CP 07340 México DF, México.*

The Wheeler-DeWitt equation is solved for some scalar-tensor theories of gravitation in the case of homogeneous and isotropic cosmological models. We present general solutions corresponding to cosmological term: $(i)\lambda(\phi) = 0$ and $(ii)\lambda(\phi) = q\phi$.

## 1 Introduction

There is a renewed interest in the scalar-tensor theories of gravitation because of the unification theories and the chaotic inflation. Therefore it seems of interest to consider the quantum cosmology originated from these theories. We begin by introducing the action integral for the scalar-tensor theory with cosmological function $\lambda(\phi)$

$$S = \frac{1}{16\pi}\int\sqrt{-g}\left\{\phi R - \frac{\omega_0}{\phi}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + 2\,\phi\,\lambda(\phi)\right\}d^4x, \tag{1}$$

using the Friedmann-Robertson-Walker metric with positive curvature, we can construct the corresponding Hamiltonian of the system. After canonical quantization we obtain the Wheeler-DeWitt equation $H\Psi(a,\phi) = 0$ for an arbitrary factor ordering, encoded in the $\alpha$ and $\beta$ parameters,

$$\left\{\frac{\omega_0}{6}\left(a^2\frac{\partial^2}{\partial a^2} + \alpha a\frac{\partial}{\partial a}\right) + a\phi\frac{\partial^2}{\partial a\partial\phi} - \left(\phi^2\frac{\partial^2}{\partial\phi^2} + \beta\phi\frac{\partial}{\partial\phi}\right) - \right.$$
$$\left. -\frac{\pi^2}{16}(2\omega_0+3)\left[3a^4\phi^2 - a^6\phi^2\lambda(\phi)\right]\right\}\Psi(a,\phi) = 0. \tag{2}$$

## 2 Solutions for the Wheeler-DeWitt equation with $\lambda(\phi) = 0$ and $\lambda(\phi) = q\phi$

We solve the WDW equation (2) by separation of variables. The corresponding solutions are show in the following cases

*i)* $k^2 > B$

$$\Psi_k(a,\phi) = \phi^{-\frac{A}{2}}a^A\sum_{\substack{m=\mp1\\ n=1,2}}\phi^{mi\sqrt{k^2\rho^2-\left(\frac{\beta-1}{2}\right)^2}}C_n H_p^{(n)}\left(\frac{3\pi}{4}ia^2\phi\right) \tag{3}$$

*ii)* $k^2 = B$

$$\Psi_k(a,\phi) = \phi^{-\frac{A}{2}} a^A (C_3 + C_4\rho\ln\phi) \sum_{n=1,2} C_n H_p^{(n)}\left(\frac{3\pi}{4} i a^2\phi\right), \tag{4}$$

*ii)* $k^2 < B$

$$\Psi_k(a,\phi) = \phi^{-\frac{A}{2}} a^A \sum_{\substack{m=\mp 1\\ n=1,2}} \phi^{mi\sqrt{\left(\frac{\beta-1}{2}\right)^2 - k^2\rho^2}} C_n H_p^{(n)}\left(\frac{3\pi}{4} i a^2\phi\right) \tag{5}$$

$k$ is a separation constant, $H_p^{(1,2)}$ are the Hankel functions of order $p$, $B = \frac{\beta-1}{2\rho}$ $A = \frac{\omega_0(\alpha-1)-3(\beta-1)}{2\omega_0+3}$, $\rho^2 = \frac{2\omega_0+3}{3}$, and $p = \sqrt{\frac{A^2}{4} - k^2}$.
If we consider an specific factor ordering $\alpha = \beta = 1$ in eq.(3), and making superposition of wave functions we have

$$\psi_k = e^{-\frac{3\pi}{4} a^2\phi \cosh[\rho\ln\phi + \mu]}. \tag{6}$$

This solution satisfied the Hawking-Page regularity condition [1], *i.e.*, *1)* the wave function is exponentially damped for large spatial geometry ($a \to \infty$), and *2)* the wave function is regular when the spatial geometry degenerates ($\Psi(a,\phi)$ does not oscillate when $a \to 0$), thus eq. (6) can be regarded like quantum wormhole solutions [2].
Now, we present solutions of WDW eq. when $\lambda(\phi) = q\phi$, where $q$ is a constant, this equation is solvable by series, but it solution haven't physical meaning. There is an interesting exact solution case with the next condition

$$\omega_0^2(\alpha-1)^2 - 6\omega_0(\alpha-1)(\beta-1) + 9(\beta-1)^2 = (1-4k^2)(2\omega_0+3)^2, \tag{7}$$

*i)* $k^2 > B$

$$\Psi_k(a,\phi) = \sum_{\substack{m=\mp 1\\ n=1,2\\ L=Ai,Bi}} \phi^{\frac{\beta-1}{2} + mi\sqrt{k^2\rho^2 - \left(\frac{\beta-1}{2}\right)^2}} C_n L\left[\frac{4}{\pi\sqrt{3q^3}}\left(3 - qa^2\phi\right)\right], \tag{8}$$

*ii)* $k^2 = B$

$$\Psi_k(a,\phi) = \phi^{\frac{\beta-1}{2}} (C_3 + C_4\rho\ln\phi) \sum_{\substack{m=1,2\\ L=Ai,Bi}} C_m L\left[\frac{4}{\pi\sqrt{3q^3}}\left(3 - qa^2\phi\right)\right], \tag{9}$$

*iii)* $k^2 < B$

$$\Psi_k(a,\phi) = \sum_{\substack{m=\mp 1\\ n=1,2\\ L=Ai,Bi}} \phi^{-\frac{A}{2} + mi\sqrt{\left(\frac{\beta-1}{2}\right)^2 - k^2\rho^2}} C_n L\left[\frac{4}{\pi\sqrt{3q^3}}\left(3 - qa^2\phi\right)\right], \tag{10}$$

where $Ai$ and $Bi$ are the Airy functions.

## 3 Final remarks

We have found solutions of WDW equation in BD theory, for two cosmological functions: $\lambda(\phi) = 0$ and $\lambda(\phi) = q\phi$, and we want to note that there are quantum WH solutions in the first case, i.e., when cosmological constant vanish, this agree with results obtained by Xiang *et al.* [3] using a particular factor ordering.


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### References


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